Bimatrix Game with Linguistic Intuitionistic Fuzzy Numbers and Its Application in Vaccine Competitors

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Abstract: In this paper, linguistic intuitionistic fuzzy numbers are introduced into the bimatrix game under quantitative conditions to better solve the uncertainty in the linguistic environment, to obtain a better strategy set in the competition between the two players. Scaling functions are used to map linguistic intuitionistic fuzzy numbers to intervals [0,1]; then, the solution methods of linear and nonlinear bi-objective programming models are established according to the Nash equilibrium theorem. Taking two vaccines in China as an example, this paper applies the model to the competition of these two vaccines in a linguistic environment and uses the bimatrix with linguistic intuitionistic fuzzy numbers to play the game.

1. Introduction

For sixty years,theory of fuzzy sets [1] has been used extensively in various fields, such as management [2], [3], ecology [4], decision-making [5-6], computers[7-8], physics [9-11], and geological engineering [12]. The intuitionistic fuzzy set is an extension of the fuzzy set. It is more maneuverable and appropriate than the traditional fuzzy set in disposing of fuzziness and indeterminacy, which was proposed by Atanassov. Xu [13] proposed definitions of uncertain linguistic variables and possibilities, as well as uncertain linguistic weighted aggregation and uncertain linguistic mixed aggregation. By challenging the concepts of linguistic fuzzy domination and Nash equilibrium, Arf [14] studied game theory based on linguistic fuzzy logic. With the continuous development of linguistic fuzzy numbers, Singh [15] developed a linguistic linear programming model and used it to settle a two-person zero-sum matrix game with the linguistic cost of binary tuples. Zhang [16] established a linguistic intuitive fuzzy set with linguistic membership and linguistic non-membership and defined several aggregation operators to aggregate linguistic intuitionistic fuzzy information. Later, Verma [17] put forward the idea of a linguistic trapezoidal fuzzy intuitionistic set in 1920 and discussed its application in multiattribute group decision-making. Verma and Agarwal [18] introduced linguistic intuitionistic fuzzy numbers into matrix games, and established a linguistic matrix game model by proposing a linguistic scale function and a new aggregation operator, taking into account the semantics of different situations.

A bimatrix game is an integral part of game theory, which is different from a matrix game. It is a non zero-sum game, that is, the sum of profits or losses of all parties in the game is not zero, which may achieve a win-win situation for two players. The fuzzy matrix game has already made significant progress [19-21]. Meanwhile, Moore [12] introduced interval fuzzy numbers into a bimatrix game. Hadik [22] discusses the invariance of support sets for interval number bimatrix games and proposes the necessary and sufficient conditions for three types of support sets to be invariant. However, the bimatrix game in which the fuzzy number is the interval number is not perfect. Yang and Li [23] studied a trapezoidal intuitive fuzzy bimatrix game with risk preference. Sakawa [4] defined the balanced solution of the bimatrix game with fuzzy benefits and proposed two methods to aggregate

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multiple fuzzy goals. Li and Lei [2] apply a new fuzzy number modeling method, the fuzzy structural element, to settle the bimatrix game with the payment value as a fuzzy variable. It is an excellent idea to apply a bimatrix game under the background of COVID-19 outbreak in Wuhan in December 2019, which occurred more than four years ago. If China's vaccines CoronaVac and CHO are taken as examples and in the absence of specific vaccination data and ambiguous environments, a linguistic intuitionistic fuzzy bimatrix game model can be built, and the mixed strategy can bring good results to other countries. To date, the literature discussed shows no study of bimatrix games in the context of linguistic intuitionistic ambiguity. Moreover, compared with the matrix game with linguistic intuitionistic fuzzy numbers, the bi-matrix game can realize the "win-win" of two players and achieve their own optimal solution. In addition, the existing methods can not solve the bimatrix game problem with qualitative information more accurately. Therefore, the main purpose of this work is to apply the bimatrix game with linguistic intuitionistic fuzzy numbers to the competitive relationship between two vaccines in China, obtain the optimal strategy set, and verify the superiority and practicability of the proposed model.

2. Linguistic Scale Function (Lsf)

The appearance of the linguistic scaling function can transform linguistic fuzzy sets into the numerical value of interval $[0,1]$ to transform linguistic fuzzy sets into traditional fuzzy sets. The four scaling functions defined in this paper have a more comprehensive range of applications. As long as they meet the continuous reversibility in the interval $[0,t]$, compared with the scaling functions defined by Wang [25], the functions defined by Wang should meet the continuous reversibility in the definition domain, and there will be fewer selectable functions.

Definition 1: Set $\hat{S}=\{s_d \mid \{d=0,1,...,t\}\}$ as a discrete linguistic term set (LTS) with cardinality of odd numbers and $\kappa_d \in [0,1]$ as a real number. The LSF φ can be defined as

$$\varphi : s_d \to \kappa_d \left(d = 0,1,2,\cdots t \right), \tag{1}$$

Where a is a strictly monotonically increasing function in the interval $[0,t]$.

The LSF φ satisfies the following conditions:

(a) $\varphi(s_0=0), \varphi(s_t)=1$;

(b) $\varphi(s_{d_1}) \geq \varphi(s_{d_2}) \Leftrightarrow d_1 \geq d_2$.

Based on the above conditions and the concavity and convexity of the function, the following four scale functions can be obtained:

a) The linguistic scale function φ_1 is a convex function.

$$\varphi_1\left(s_d\right)=\kappa_d=\sin\left(\frac{d\pi}{2t}\right), d=0,1,\cdots,t. \tag{2}$$

b) The linguistic scale function φ_2 is a concave function.

$$\varphi_2\left(s_d\right)=\kappa_d=\left(\frac{d}{t}\right)^3, d=0,1,\cdots,t. \tag{3}$$

c) The linguistic scale function φ_3 is concave in interval $[0,\frac{t}{2}]$ and convex in interval $[\frac{t}{2},t]$.

$$\varphi_3^*\left(s_d\right)=\kappa_d=\begin{cases}\dfrac{d(\varsigma^{\frac{t}{2}}-\varsigma^{\frac{t}{2}-d})}{t\times(\varsigma^{\frac{t}{2}}-1)}, d=0,1,\cdots,\dfrac{t}{2}\\ \dfrac{(d-\frac{t}{2})\varsigma^{t-d}+\frac{t}{2}}{t}, d=\dfrac{t}{2},\cdots,t.\end{cases} \tag{4}$$

$\varsigma > 1$ is a threshold value,which can be determined subjectively depending on the particular problem. In this paper, we take $\varsigma = \sqrt[t+2]{t}$.

d)The linguistic scale function φ_4 is convex in the interval $[0,\frac{t}{2}]$ and concave in the interval $[\frac{t}{2},t]$.

$$\varphi_4^*(s_d)=\kappa_d=\begin{cases}\dfrac{t^\rho-(t-d)^\rho}{2\times(t^\rho-\frac{t}{2}^\rho)}, d=0,1,\cdots,\frac{t}{2}\\ \dfrac{d^\rho+t^\rho-2\times\frac{t}{2}^\rho}{2\times(t^\rho-\frac{t}{2}^\rho)}, d=\frac{t}{2},\cdots,t\end{cases}, \quad (5)$$

Where $\rho\geq 1$ are determined according to the particular problem. We take $\rho=\frac{t}{2}$.

To guarantee the integrity of the information during the calculation, the LSF can be further extended to the following forms of extended continuous LTS:

Definition 2: Set $\hat{S}=\{s_d \mid s_0\leq s_d\leq s_t, d\in[0,t](t\ is\ even)\}$ as an extended continuous LTS and $\kappa_d\in[0,1]$ as a real number. The linguistic scale function φ^* can be defined as:

$$\varphi^*:\hat{S}_{[0,t]}\to\kappa_d,$$

Where φ^* is a strictly monotonic increasing and continuous function in $[0,t]$.then, its inverse function is defined as φ^{*-1}.

3. Linguistic Intuitionistic Fuzzy Bimatrix Game and Its Solution (Lsf)

A bimatrix game with linguistic intuitionistic fuzzy numbers refers to a bimatrix game in which each player's strategy is described in linguistic terms. Expressed in mathematical symbols, $\mathbb{S}^m=(\alpha_1,\alpha_2,...,\alpha_m)$ and $\mathbb{S}^n=(\beta_1,\beta_2,...,\beta_m)$ represent the pure strategy set of player 1 and player 2 respectively. $\hat{S}_{[0,t]}=\{s_d \mid s_0\leq s_d\leq s_t, d\in[0,t]\}$ indicate a continuous linguistic term set with cardinality of odd numbers, where t is a positive integer. When player 1 selects strategy $\alpha_i\in\mathbb{S}^m$ and player 2 selects strategy $\beta_j\in\mathbb{S}^n$, the payoff values of player 1 and player 2 are also expressed as linguistic intuitionistic fuzzy numbers (LIFNs): $\Upsilon_{ij}^{\mathcal{A}}=<s_{\theta_{ij}^{\mathcal{A}}},s_{\delta_{ij}^{\mathcal{A}}}>$, $\Upsilon_{ij}^{\mathcal{B}}=<s_{\theta_{ij}^{\mathcal{B}}},s_{\delta_{ij}^{\mathcal{B}}}>$, where $s_{\theta_{ij}^{\mathcal{A}}}$, $s_{\delta_{ij}^{\mathcal{A}}}$, $s_{\theta_{ij}^{\mathcal{B}}}$, $s_{\delta_{ij}^{\mathcal{B}}}\in\hat{S}_{[0,t]}$,and that $0\leq\theta_{ij}^{\mathcal{A}}+\delta_{ij}^{\mathcal{A}}\leq t$, $0\leq\theta_{ij}^{\mathcal{B}}+\delta_{ij}^{\mathcal{B}}\leq t$. Therefore, the payment matrix of player 1 and player 2 can be expressed as follows:

$$\mathcal{A}=\left(\Upsilon_{ij}^{\mathcal{A}}\right)_{m\times n}\begin{pmatrix}\left\langle s_{\theta_{11}^{\mathcal{A}}},s_{\delta_{11}^{\mathcal{A}}}\right\rangle & \cdots & \left\langle s_{\theta_{1n}^{\mathcal{A}}},s_{\delta_{1n}^{\mathcal{A}}}\right\rangle\\ \left\langle s_{\theta_{21}^{\mathcal{A}}},s_{\delta_{21}^{\mathcal{A}}}\right\rangle & \cdots & \left\langle s_{\theta_{2n}^{\mathcal{A}}},s_{\delta_{2n}^{\mathcal{A}}}\right\rangle\\ \vdots & & \vdots\\ \left\langle s_{\theta_{m1}^{\mathcal{A}}},s_{\delta_{m1}^{\mathcal{A}}}\right\rangle & \cdots & \left\langle s_{\theta_{mn}^{\mathcal{A}}},s_{\delta_{mn}^{\mathcal{A}}}\right\rangle\end{pmatrix},\qquad \mathcal{B}=\left(\Upsilon_{ij}^{\mathcal{B}}\right)_{m\times n}\begin{pmatrix}\left\langle s_{\theta_{11}^{\mathcal{B}}},s_{\delta_{11}^{\mathcal{B}}}\right\rangle & \cdots & \left\langle s_{\theta_{1n}^{\mathcal{B}}},s_{\delta_{1n}^{\mathcal{B}}}\right\rangle\\ \left\langle s_{\theta_{21}^{\mathcal{B}}},s_{\delta_{21}^{\mathcal{B}}}\right\rangle & \cdots & \left\langle s_{\theta_{2n}^{\mathcal{B}}},s_{\delta_{2n}^{\mathcal{B}}}\right\rangle\\ \vdots & & \vdots\\ \left\langle s_{\theta_{m1}^{\mathcal{B}}},s_{\delta_{m1}^{\mathcal{B}}}\right\rangle & \cdots & \left\langle s_{\theta_{mn}^{\mathcal{B}}},s_{\delta_{mn}^{\mathcal{B}}}\right\rangle\end{pmatrix}.$$

The mixed strategy is expressed as a vector $x=(x_1,x_2,\cdots,x_m)^T$ and $y=(y_1,y_2,\cdots,y_n)^T$, where x_i and y_j represent the probability of selecting pure strategies α_i,β_j for player 1 and player 2. They satisfy $\sum_{i=1}^m x_i=1$, $\sum_{j=1}^n y_j=1$ .Therefore, $X=\left\{x\middle|x^Te_m=1,x\geq 0\right\}$, $Y=\left\{y\middle|y^Te_n=1,y\geq 0\right\}$ can be used to represent the mixed strategy sets of player 1 and player 2. Therefore, the bimatrix game with linguistic intuitionistic fuzzy numbers can be expressed as $\mathbb{G}=\left(\mathbb{S}^m,X,\mathbb{S}^n,Y,\hat{S}_{[0,t]},\mathcal{A},\mathcal{B}\right)$ Bimatrix game $(\mathcal{A},\mathcal{B})$ for arbitrary linguistic intuitionistic fuzzy sets. When player 1 chooses mixed strategy $x\in X$ and player 2 chooses mixed strategy $y\in Y$, their expected payoffs can be calculated as Eqs 6 and Eqs 7:

$$E_{\mathcal{A}}=x^T\mathcal{A}y, \quad (6)$$

$$E_{\mathcal{B}}=x^T\mathcal{B}y. \quad (7)$$

Definition 3: $\forall x\in X$ and $\forall y\in Y$, if $(x^*,y^*)\in X\times Y$ exists, it meets the following conditions:

$$x^{*}T\mathcal{A}y^{*} \geq x^{T}\mathcal{A}y^{*},$$
$$x^{*T}\mathcal{B}y^{*} \geq x^{*T}\mathcal{B}y. \tag{8}$$

It can be considered that (x^{*}, y^{*}) is the Nash equilibrium solution on the mixed strategy of the bimatrix $(\mathcal{A},\mathcal{B})$ with a linguistic intuitionistic fuzzy set. x^{*} and y^{*} are considered the optimal strategies of player 1 and player 2, respectively. $\Psi_{\mathcal{A}}^{*} = \left(s_{\theta_{\Psi_{\mathcal{A}}^{*}}}, s_{\delta_{\Psi_{\mathcal{A}}^{*}}}\right)$ and $\Psi_{\mathcal{B}}^{*} = \left(s_{\theta_{\Psi_{\mathcal{B}}^{*}}}, s_{\delta_{\Psi_{\mathcal{B}}^{*}}}\right)$ are known as the game payoffs of player 1 and player 2. Especially in the case of the pure strategy, expressed on the basis of Def.3 as: for pure strategy $\alpha_i \in \mathbb{S}^m$ and $\beta_j \in \mathbb{S}^n$, if $(\alpha_i^{*}, \beta_j^{*})$ meets:

$$\begin{cases} s_{\theta_{ij^*}^{\mathcal{A}}} \leq s_{\theta_{i^*j^*}^{\mathcal{A}}} \\ s_{\delta_{ij^*}^{\mathcal{A}}} \geq s_{\delta_{i^*j^*}^{\mathcal{A}}} \end{cases}, (i = 1, 2, \cdots, m), \qquad \begin{cases} s_{\theta_{i^*j}^{\mathcal{B}}} \leq s_{\theta_{i^*j^*}^{\mathcal{B}}} \\ s_{\delta_{i^*j}^{\mathcal{B}}} \geq s_{\delta_{i^*j^*}^{\mathcal{B}}} \end{cases}, (j = 1, 2, \cdots, n).$$

Then, it can be considered that $(\alpha_i^{*}, \beta_j^{*})$ is the Nash equilibrium solution on the pure strategy of the bimatrix $(\mathcal{A},\mathcal{B})$ with a linguistic intuitionistic fuzzy set.

Let $\Re_{\mathcal{A}}^{\theta} = \ln\left(1 - \varphi^{*}\left(s_{\theta_{\Psi_{\mathcal{A}}^{*}}}\right)\right)$ and $\Re_{\mathcal{A}}^{\delta} = \ln\left(s_{\delta_{\Psi_{\mathcal{A}}^{*}}}\right)$, utilizing the weighted average operator proposed by Harsanyi [26], since the strategy set X is a finite compact convex set, according to the definition of convex function, the extremum only exists at the pole. Let $\gamma\Re_{\mathcal{A}}^{\theta} + (1-\gamma)\Re_{\mathcal{A}}^{\delta} = \Re_{\mathcal{A}}$, the mixed strategy x^{*} and expected payoffs of player 1 can be obtained by solving the following nonlinear bi-objective programming model:

(MOD1) $\max\{\Re_{\mathcal{A}}\}$

$$s.t.\begin{cases} \sum_{j=1}^{n} \gamma y_j \left(\ln\left(1 - \varphi^{*}\left(s_{\theta_{ij}^{\mathcal{A}}}\right)\right)\right) + (1-\gamma) y_j \left(\ln\left(\varphi^{*}\left(s_{\delta_{ij}^{\mathcal{A}}}\right)\right)\right) \geq \Re_{\mathcal{A}} \\ s_{\theta_{ij}^{\mathcal{A}}}, s_{\delta_{ij}^{\mathcal{A}}} \in \hat{S}_{[0,t]}, \\ 0 \leq \theta_{ij}^{\mathcal{A}} + \delta_{ij}^{\mathcal{A}} \leq t, \\ \sum_{j=1}^{n} y_j = 1, y_j \geq 0, j = 1, 2, \cdots, n. \end{cases} \tag{9}$$

Similarly, let $\Re_{\mathcal{B}}^{\theta} = \ln\left(1 - \varphi^{*}\left(s_{\theta_{\Psi_{\mathcal{B}}^{*}}}\right)\right)$, $\Re_{\mathcal{B}}^{\delta} = \ln\left(s_{\delta_{\Psi_{\mathcal{B}}^{*}}}\right)$, utilizing the weighted average operator proposed by Harsanyi [26]. Let $\omega\Re_{\mathcal{B}}^{\theta} + (1-\omega)\Re_{\mathcal{B}}^{\delta} = \Re_{\mathcal{B}}$, the mixed strategy y^{*} and expected payoffs of player 2 can be obtained by solving the following nonlinear bi-objective programming model:

(MOD2) $\max\{\Re_{\mathcal{B}}\}$

$$s.t.\begin{cases} \sum_{i=1}^{m} \omega x_i \left(\ln\left(1 - \varphi^{*}\left(s_{\theta_{ij}^{\mathcal{B}}}\right)\right)\right) + (1-\omega) x_i \left(\ln\left(\varphi^{*}\left(s_{\delta_{ij}^{\mathcal{B}}}\right)\right)\right) \geq \Re_{\mathcal{B}} \\ s_{\theta_{ij}^{\mathcal{B}}}, s_{\delta_{ij}^{\mathcal{B}}} \in S_{[0,t]}, \\ 0 \leq \theta_{ij}^{\mathcal{B}} + \delta_{ij}^{\mathcal{B}} \leq t, \\ \sum_{i=1}^{m} x_i = 1, x_i \geq 0, i = 1, 2, \cdots, m. \end{cases} \tag{10}$$

Then $(x^{*}, \Re_{\mathcal{A}}^{*})$ and $(y^{*}, \Re_{\mathcal{B}}^{*})$ are the optimals of (MOD1) and (MOD2).

4. Application

At present, the two COVID-19 vaccines in China are CoronaVac and CHO, with the former being an inactivated vaccine and the latter a recombinant protein vaccine. The vaccination rate is closely related to the government's advocacy, the innovation of enterprises, and the severity of the epidemic. Since there are no more than two of the COVID-19 vaccines commercially available, the increased uptake of one does not implies a decreased uptake of the other, in which case finding an optimal strategy to maximize uptake of both vaccines is desirable. Due to limited information and unpredictability of the market, there is no daily vaccination population per vaccine on the network, which means that we need to use linguistic intuitionistic fuzzy bimatrix game to find the optimal strategy. The applicability of (MOD1) and (MOD2) is demonstrated by taking these two vaccines as examples.

First, we need to obtain linguistic data and obtain the relevant comments of the people on the two vaccines from March 1, 2021 to February 28, 2022 from Weibo. Then, data cleaning was performed to screen out irrelevant text and remove special symbols, spaces and unrelated microblog expressions. Then, the text was segmented using a word-breaking package and labeled manually. Finally, the processed data were input into the model to obtain the fuzzy linguistic payment matrix of CoronaVac $(\mathcal{A})$ and CHO $(\mathcal{B})$. Its linguistic fuzzy number terminology set is shown in Table 1.

$$\mathcal{A}=\begin{pmatrix} \langle s_5,s_1\rangle & \langle s_3,s_1\rangle & \langle s_4,s_3\rangle \\ \langle s_6,s_2\rangle & \langle s_4,s_2\rangle & \langle s_5,s_1\rangle \\ \langle s_4,s_2\rangle & \langle s_2,s_1\rangle & \langle s_5,s_1\rangle \end{pmatrix},\qquad \mathcal{B}=\begin{pmatrix} \langle s_3,s_2\rangle & \langle s_5,s_3\rangle & \langle s_4,s_2\rangle \\ \langle s_2,s_4\rangle & \langle s_3,s_2\rangle & \langle s_3,s_2\rangle \\ \langle s_3,s_2\rangle & \langle s_4,s_2\rangle & \langle s_3,s_4\rangle \end{pmatrix}.$$

Table 1 Nine linguistic labels and their corresponding semantic meanings.

linguistic labels	semantic meanings
$s0$	very low (VL)
$s1$	low (L)
$s2$	moderately low (ML)
$s3$	slightly low (SL)
$s4$	average (Avg)
$s5$	slightly high (SH)
$s6$	moderately high (MH)
$s7$	high (H)
$s8$	very high (VH)

The results in the Tab.2, 3, 4, 5 show that when the values of λ and ω are small, government propaganda can significantly improve the vaccination rate, and combined with the impact of the epidemic environment, the maximum vaccination rate can be achieved. When the values of λ and ω are large, the vaccination rate can only be improved if the enterprise itself is highly innovative, but the effect of enterprise innovation is not as good as that of the other two strategies. When the epidemic is not serious, the publicity of the government and the innovation of the enterprise itself can improve the vaccination rate. When the epidemic is serious, even if the government does not carry out publicity, people will take the initiative to vaccinate to avoid infection. Therefore, when we choose the appropriate strategy and adjust the strategy to the environment in time, we can continuously improve the vaccination rate.

5. Conclusion

In this paper, the linguistic intuitionistic fuzzy number is used to represent the uncertainty of the player's payment value in the linguistic environment. Considering that the two players have different weights in the market, the weighted average index, λ, ω, is introduced to construct a bimatrix game model with linguistic intuitionistic fuzzy numbers. The linguistic intuitionistic fuzzy number is mapped to the interval [0,1] by using the scaling function, and the optimal policy set is obtained by solving the linear or nonlinear biobjective programming twice. The method applied in China's two vaccines in the game, in the future, can also be used with different linguistic term sets of bimatrix games applied to vaccine competition, such as bimatrix game with hesitant fuzzy sets and linguistic term sets with a probability of bimatrix games, which can further research more effectively for solving bimatrix games with linguistic term sets. In future work, we will continue to supplement the shortcomings of the model and introduce more linguistic term sets into the bimatrix game, such as the hesitation fuzzy linguistic term set and probability linguistic term set.

$$\max\{\Re_A\}$$

$$s.t.\begin{cases}\left[\gamma\ln\left(1-\varphi^*(s_5)\right)+(1-\gamma)\ln\left(\varphi^*(s_1)\right)\right]y_1+\left[\gamma\ln\left(1-\varphi^*(s_3)\right)+(1-\gamma)\ln\left(\varphi^*(s_1)\right)\right]y_2\\ \qquad+\left[\gamma\ln\left(1-\varphi^*(s_4)\right)+(1-\gamma)\ln\left(\varphi^*(s_3)\right)\right]y_3\geq\Re_A,\\ \left[\gamma\ln\left(1-\varphi^*(s_6)\right)+(1-\gamma)\ln\left(\varphi^*(s_2)\right)\right]y_1+\left[\gamma\ln\left(1-\varphi^*(s_4)\right)+(1-\gamma)\ln\left(\varphi^*(s_2)\right)\right]y_2\\ \qquad+\left[\gamma\ln\left(1-\varphi^*(s_5)\right)+(1-\gamma)\ln\left(\varphi^*(s_1)\right)\right]y_3\geq\Re_A,\\ \left[\gamma\ln\left(1-\varphi^*(s_4)\right)+(1-\gamma)\ln\left(\varphi^*(s_2)\right)\right]y_1+\left[\gamma\ln\left(1-\varphi^*(s_2)\right)+(1-\gamma)\ln\left(\varphi^*(s_1)\right)\right]y_2\\ \qquad+\left[\gamma\ln\left(1-\varphi^*(s_5)\right)+(1-\gamma)\ln\left(\varphi^*(s_1)\right)\right]y_3\geq\Re_A,\\ \sum_{j=1}^{3}y_j=1,y_j\geq0,j=1,2,3.\end{cases}\tag{11}$$

$$\max\{\Re_B\}$$

$$s.t.\begin{cases}\left[\omega\ln\left(1-\varphi^*(s_3)\right)+(1-\omega)\ln\left(\varphi^*(s_2)\right)\right]x_1+\left[\omega\ln\left(1-\varphi^*(s_2)\right)+(1-\omega)\ln\left(\varphi^*(s_4)\right)\right]x_2\\ \qquad+\left[\omega\ln\left(1-\varphi^*(s_3)\right)+(1-\omega)\ln\left(\varphi^*(s_2)\right)\right]x_3\geq\Re_B,\\ \left[\omega\ln\left(1-\varphi^*(s_5)\right)+(1-\omega)\ln\left(\varphi^*(s_3)\right)\right]x_1+\left[\omega\ln\left(1-\varphi^*(s_3)\right)+(1-\omega)\ln\left(\varphi^*(s_2)\right)\right]x_2\\ \qquad+\left[\omega\ln\left(1-\varphi^*(s_4)\right)+(1-\omega)\ln\left(\varphi^*(s_2)\right)\right]x_3\geq\Re_B,\\ \left[\omega\ln\left(1-\varphi^*(s_4)\right)+(1-\omega)\ln\left(\varphi^*(s_2)\right)\right]x_1+\left[\omega\ln\left(1-\varphi^*(s_3)\right)+(1-\omega)\ln\left(\varphi^*(s_2)\right)\right]x_2\\ \qquad+\left[\omega\ln\left(1-\varphi^*(s_3)\right)+(1-\omega)\ln\left(\varphi^*(s_4)\right)\right]x_3\geq\Re_B,\\ \sum_{i=1}^{3}x_i=1,x_i\geq0,i=1,2,3.\end{cases}\tag{12}$$

References

[1] L. A. Zadeh, "Fuzzy sets," Information and Control, vol.8, no.3, pp. 338-353, 1965.

[2] C. Li and T.Lei, "Fuzzy Bi-matrix games based on fuzzy structured element," 2017 13th International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD). IEEE, 2018.

[3] H. L. Lu, B. B. Sun and Z. H. Hu, "Research on Energy Management Strategy of Battery-Flywheel Hybrid Energy Storage Electric Vehicle," IAENG International Journal of Computer Science, vol. 49, no. 4, 2022.

[4] Nishizaki and M. Sakawa, "Equilibrium solutions in multiobjective bimatrix games with fuzzy payoffs and fuzzy goals," Fuzzy Sets and Systems, vol.111, no.1, pp. 99-116, 2000.

[5] L.Y. Zhang, C. L. Liang and Z.Y.Yang, "Research on the emergency decision-making method based on case-based reasoning under triangle fuzzy preference," IAENG International Journal of Computer Science, vol. 49, no.1, pp. 11-18, 2022.

[6] W. J. Sun, C.F. Zhu and K. R. Liu, "Prospect Selection Decisions for Emergency Logistics Paths in Fuzzy Environments," IAENG International Journal of Applied Mathematics, vol.53, no.1, pp. 1-11, 2023.

[7] H. X. Guan, B.Yong and H. R. Wang, "Multiple faults diagnosis of distribution network lines based on convolution neural network with fuzzy optimization," IAENG International Journal of Computer Science, vol. 47, no. 3, pp. 567-571, 2020.

[8] M. Blondeel, S. Schockaert and D. Vermeir, "Complexity of fuzzy answer set programming under Lukasiewicz semantics," International journal of approximate reasoning, vol.55, no.9, pp. 1971-2003, 2014.

[9] S. Panda, K. D. Jayanta and G. B. Panda, “Stochastic Differential Equation With Fuzzy Coefficients,” IAENG International Journal of Applied Mathematics, vol.53, no.1, pp. 1-10, 2023.

[10] V.F. Kravchenko and VIPI. Pustovot, “Three-dimensional filtration of multichannel video sequences on the basis of fuzzy-set theory,” Doklady Physics, 2010.

[11] M.Wang,X.Wang and Q. Liu, “A novel multi-dimensional cloud model coupled with connection numbers theory for evaluation of slope stability,” Applied Mathematical Modelling, vol.77, no.1, pp. 426-438, 2020.

[12] R.Pramanik,D.K. Baidya and N. Dhang, “Reliability assessment of three-dimensional bearing capacity of shallow foundation using fuzzy set theory,” Frontiers of Structural and Civil Engineering, vol. 15, pp. 478-489, 2021.

[13] Z. S. Xu, “Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment,” Information Sciences, vol.168, pp. 171-184, 2004.

[14] B.Arf, “Linguistic fuzzy-logic game theory,” Journal of conflict resolution, vol.50, no.1, pp. 28–57,2006.

[15] Singh, A. Gupta and A. Mehra, “Matrix games with 2-tuple linguistic information,” Annals of Operations Research, 2018.

[16] H.Zhang, “Linguistic Intuitionistic Fuzzy Sets and Application in MAGDM,” Journal of applied mathematics, vol. 2014, pp. 1-11, 2014.

[17] R.Verma, “On aggregation operators for linguistic trapezoidal fuzzy intuitionistic fuzzy sets and their application to multiple attribute group decision making,” Journal of Intelligent and Fuzzy Systems, vol. 38, no.3, pp. 2907-2950, 2020.

[18] R.Verma and A. Aggarwal, “Matrix games with linguistic intuitionistic fuzzy Payoffs: Basic results and solution methods,” Artificial Intelligence Review, vol.54, no.7, pp. 5127-5162, 2021.

[19] D.Dubois and H. Prada, “Fuzzy sets and systems: theory and applications,” Academic Press,1980.

[20] S. A. Orlovski, “Fuzzy Goals and Sets of Choices in Two-Person Games,” Springer Netherlands,1990.

[21] V. Vijay, S. Chandra and C. R. Bector, “Matrix games with fuzzy goals and fuzzy payoffs,” Omega, vol.33, no. 5, pp. 425-429, 2005.

[22] M. Hladik, “Multiparametric linear programming: Support set and optimal partition invariancy,” European Journal of Operational Research, vol. 202, no.1, pp. 25-31, 2010.

[23] J. Yang, D. F. Li and L. B. Lai, “Trapezoidal intuitionistic fuzzy bimatrix game model with risk preference and its solving method,”Applied Mathematics Journal of Chinese Universities(Ser.A), vol. 31, no.3, pp.357-365, 2016.

[24] Z. Xu, “Intuitionistic Fuzzy Aggregation and Clustering,” Studies in Fuzziness and Soft Computing, 2012.

[25] J. Q. Wang, J. T. Wu and J. Wang, “Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems,” Information Sciences, vol.288, pp. 55-72, 2014.

[26] J.C.Harsanyi, “Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility,” Journal of Political Economy, vol.63, no.4, pp. 309–321,1955.

Table 2 Optimal solutions and the corresponding expected payoffs of the models given in Eqs.11 and 12 with $\varphi^* = \varphi_1^*$.

$\varphi^* = \varphi_1^*$					
$\langle \lambda, \omega \rangle$	y^{*T}	x^{*T}	$\langle \Re_A, \Re_B \rangle$	Ψ_A^*	Ψ_B^*
(0.1, 0.1)	(0.5194, 0.1316, 0.3490)	(0.5419, 0.2026, 0.2553)	(-1.2881, -0.8270)	(s4.4351, s1.4064)	(s2.9086, s2.9209)
(0.1, 0.4)	(0.5194, 0.1316, 0.3490)	(0, 1, 0)	(-1.2881, -0.9007)	(s2.8340, s1.0941)	(s4.4584, s2.4599)
(0.1, 0.7)	(0.5194, 0.1316, 0.3490)	(0, 1, 0)	(-1.2881, -0.8558)	(s2.8340, s1.0941)	(s4.4584, s2.4599)
(0.1, 0.9)	(0.5194, 0.1316, 0.3490)	(0, 1, 0)	(-1.2881, -0.8259)	(s2.8340, s1.0941)	(s4.4584, s2.4599)
(0.4, 0.1)	(0, 0.8659, 0.1341)	(0.5419, 0.2026, 0.2553)	(-1.2431, -0.8270)	(s5.2932, s1.6395)	(s2.5922, s2.7932)
(0.4, 0.4)	(0, 0.8659, 0.1341)	(0, 1, 0)	(-1.2431, -0.9007)	(s3.7655, s1.8302)	(s3.1364, s2.0126)
(0.4, 0.7)	(0, 0.8659, 0.1341)	(0, 1, 0)	(-1.2431, -0.8558)	(s3.7655, s1.8302)	(s3.1364, s2.0126)
(0.4, 0.9)	(0, 0.8659, 0.1341)	(0, 1, 0)	(-1.2431, -0.8259)	(s3.7655, s1.8302)	(s3.1364, s2.0126)
(0.7, 0.1)	(0, 1, 0)	(0.5419, 0.2026, 0.2553)	(-1.1477, -0.827)	(s5.4518, s1.6711)	(s2.4854, s2.8681)
(0.7, 0.4)	(0, 1, 0)	(0, 1, 0)	(-1.1477, -0.9007)	(s4, s2)	(s3, s2)
(0.7, 0.7)	(0, 1, 0)	(0, 1, 0)	(-1.1477, -0.8558)	(s4, s2)	(s3, s2)
(0.7, 0.9)	(0, 1, 0)	(0, 1, 0)	(-1.1477, -0.8259)	(s4, s2)	(s3, s2)
(0.9, 0.1)	(0, 1, 0)	(0.5419, 0.2026, 0.2553)	(-1.2012, -0.827)	(s5.4518, s1.6711)	(s2.4854, s2.8681)
(0.9, 0.4)	(0, 1, 0)	(0, 1, 0)	(-1.2012, -0.9007)	(s4, s2)	(s3, s2)
(0.9, 0.7)	(0, 1, 0)	(0, 1, 0)	(-1.2012, -0.8558)	(s4, s2)	(s3, s2)
(0.9, 0.9)	(0, 1, 0)	(0, 1, 0)	(-1.2012, -0.8259)	(s4, s2)	(s3, s2)

Table 3 Optimal solutions and the corresponding expected payoffs of the models given in Eqs.11 and 12 with $\varphi^* = \varphi_2^*$.

$\varphi^* = \varphi_2^*$					
$\langle \lambda, \omega \rangle$	y^{*T}	x^{*T}	$\langle \Re_A, \Re_B \rangle$	Ψ_A^*	Ψ_B^*
(0.1, 0.1)	(0.6042, 0.0135, 0.3823)	(0.4664, 0.2655, 0.2681)	(-4.5025, -3.2504)	(s4.2958, s1.3614)	(s3.7752, s2.3018)
(0.1, 0.4)	(0.6042, 0.0135, 0.3823)	(0.4956, 0.2444, 0.2600)	(-4.5025, -2.2083)	(s4.3287, s1.3646)	(s3.7328, s2.2854)
(0.1, 0.7)	(0.6042, 0.0135, 0.3823)	(0.5983, 0.1705, 0.2312)	(-4.5025, -1.1746)	(s4.4387, s1.3762)	(s3.5736, s2.2298)
(0.1, 0.9)	(0.6042, 0.0135, 0.3823)	(0, 1, 0)	(-4.5025, -0.4646)	(s2.7300, s1.0094)	(s4.6644, s2.5552)
(0.4, 0.1)	(0.5616, 0.0776, 0.3608)	(0.4664, 0.2655, 0.2681)	(-3.1132, -3.2504)	(s4.3923, s1.3793)	(s3.7201, s2.3300)
(0.4, 0.4)	(0.5616, 0.0776, 0.3608)	(0.4956, 0.2444, 0.2600)	(-3.1132, -2.2083)	(s4.3754, s1.4555)	(s3.6783, s2.3175)
(0.4, 0.7)	(0.5616, 0.0776, 0.3608)	(0.5983, 0.1705, 0.2311)	(-3.1132, -1.1746)	(s4.4982, s1.4612)	(s3.4419, s2.4264)
(0.4, 0.9)	(0.5616, 0.0776, 0.3608)	(0, 1, 0)	(-3.1132, -0.4646)	(s2.8542, s1.0553)	(s4.5917, s2.5114)
(0.7, 0.1)	(0.4515, 0.2419, 0.3065)	(0.4664, 0.2655, 0.2681)	(-1.6947, -3.2504)	(s4.6173, s1.4262)	(s3.5686, s2.4038)
(0.7, 0.4)	(0.4515, 0.2419, 0.3065)	(0.4956, 0.2444, 0.2600)	(-1.6947, -2.2083)	(s4.6539, s1.4312)	(s3.5299, s2.4022)
(0.7, 0.7)	(0.4515, 0.2419, 0.3065)	(0.5983, 0.1705, 0.2312)	(-1.6947, -1.1746)	(s4.7765, s1.4490)	(s3.3802, s2.3963)
(0.7, 0.9)	(0.4515, 0.2419, 0.3065)	(0, 1, 0)	(-1.6947, -0.4646)	(s3.1273, s1.1828)	(s4.3884, s2.4022)
(0.9, 0.1)	(0, 0.9303, 0.0697)	(0.4664, 0.2655, 0.2681)	(-0.6546, -3.2504)	(s5.3292, s1.6397)	(s2.6948, s2.7370)
(0.9, 0.4)	(0, 0.9303, 0.0697)	(0.4956, 0.2444, 0.2600)	(-0.6546, -2.2083)	(s5.3739, s1.6485)	(s2.6683, s2.7879)
(0.9, 0.7)	(0, 0.9303, 0.0697)	(0.5983, 0.1705, 0.2312)	(-0.6546, -1.1746)	(s5.5139, s1.6899)	(s2.5704, s2.9746)
(0.9, 0.9)	(0, 0.9303, 0.0697)	(0, 1, 0)	(-0.6546, -0.4646)	(s3.9215, s1.9057)	(s3.0960, s2.0)

Table 4 Optimal solutions and the corresponding expected payoffs of the models given in Eqs.11 and 12 with $\varphi^* = \varphi_3^*$.

$\varphi^* = \varphi_3^*$					
$\langle \lambda, \omega \rangle$	y^{*T}	x^{*T}	$\langle \Re_A, \Re_B \rangle$	Ψ_A^*	Ψ_B^*
(0.1, 0.1)	(0.5624, 0.0692, 0.3684)	(0.4974, 0.2415, 0.2610)	(-2.3068, -1.4797)	(s4.3400, s1.3740)	(s3.6569, s2.3040)
(0.1, 0.4)	(0.5624, 0.0692, 0.3684)	(0.7939, 0.02986, 0.1763)	(-2.3068, -1.2629)	(s4.6501, s1.4126)	(s3.1521, s2.1808)
(0.1, 0.7)	(0.5624, 0.0692, 0.3684)	(0, 1, 0)	(-2.3068, -0.8270)	(s2.9274, s1.0473)	(s4.5481, s2.5061)
(0.1, 0.9)	(0.5624, 0.0692, 0.3684)	(0, 1, 0)	(-2.3068, -0.5202)	(s2.9274, s1.0473)	(s4.5481, s2.5061)
(0.4, 0.1)	(0.3653, 0.3570, 0.2777)	(0.4974, 0.2415, 0.2610)	(-1.8991, -1.4797)	(s4.6620, s1.4588)	(s3.3555, s2.4212)
(0.4, 0.4)	(0.3653, 0.3570, 0.2777)	(0.7939, 0.02986, 0.1763)	(-1.8991, -1.2629)	(s5.0251, s1.5215)	(s2.9036, s2.5117)
(0.4, 0.7)	(0.3653, 0.3570, 0.2777)	(0, 1, 0)	(-1.8991, -0.8270)	(s3.2216, s1.2764)	(s4.1859, s2.3137)
(0.4, 0.9)	(0.3653, 0.3570, 0.2777)	(0, 1, 0)	(-1.8991, -0.5202)	(s3.2216, s1.2764)	(s4.2410, s2.2663)
(0.7, 0.1)	(0, 0.8716, 0.1280)	(0.4974, 0.2415, 0.2610)	(-1.1663, -1.4797)	(s5.1870, s1.6288)	(s2.6676, s2.7447)
(0.7, 0.4)	(0, 0.8716, 0.1280)	(0.7939, 0.02986, 0.1763)	(-1.1663, -1.2629)	(s5.6166, s1.7615)	(s2.3667, s3.2605)
(0.7, 0.7)	(0, 0.8716, 0.1280)	(0, 1, 0)	(-1.1663, -0.8270)	(s3.8277, s1.8256)	(s3.1567, s2)
(0.7, 0.9)	(0, 0.8716, 0.1280)	(0, 1, 0)	(-1.1663, -0.5202)	(s3.8277, s1.8256)	(s3.1567, s2)
(0.9, 0.1)	(0, 1, 0)	(0.4974, 0.2415, 0.2610)	(-0.8139, -1.4797)	(s5.3338, s1.6337)	(s2.5600, s2.8119)
(0.9, 0.4)	(0, 1, 0)	(0.7939, 0.02986, 0.1763)	(-0.8139, -1.2629)	(s5.7938, s1.7650)	(s2.2486, s3.4445)
(0.9, 0.7)	(0, 1, 0)	(0, 1, 0)	(-0.8139, -0.8270)	(s4, s2)	(s3, s2)
(0.9, 0.9)	(0, 1, 0)	(0, 1, 0)	(-0.8139, -0.5202)	(s4, s2)	(s3, s2)

Table 5 Optimal solutions and the corresponding expected payoffs of the models given in Eqs.11 and 12 with $\varphi^* = \varphi_4^*$.

$\varphi^* = \varphi_4^*$					
$\langle \lambda, \omega \rangle$	y^{*T}	x^{*T}	$\langle \Re_A, \Re_B \rangle$	Ψ_A^*	Ψ_B^*
(0.1, 0.1)	(0.5296, 0.0581, 0.4123)	(0.5267, 0.2205, 0.2529)	(-1.1846, -0.8995)	(s4.2362, s1.3383)	(s3.2050, s2.2409)
(0.1, 0.4)	(0.5296, 0.0581, 0.4123)	(0.7721, 0.0308, 0.1971)	(-1.1846, -0.8337)	(s4.6813, s1.3962)	(s2.9597, s2.1497)
(0.1, 0.7)	(0.5296, 0.0581, 0.4123)	(0.0959, 0.7857, 0.1184)	(-1.1846, -0.7197)	(s2.9051, s1.1200)	(s4.2159, s2.4035)
(0.1, 0.9)	(0.5296, 0.0581, 0.4123)	(0.1331, 0.6836, 0.1833)	(-1.1846, -0.6561)	(s3.0794, s1.1647)	(s4.0060, s2.3929)
(0.4, 0.1)	(0.3140, 0.3411, 0.3449)	(0.5267, 0.2205, 0.2529)	(-1.0401, -0.8995)	(s4.7347, s1.4305)	(s2.9022, s2.3699)
(0.4, 0.4)	(0.3140, 0.3411, 0.3449)	(0.7721, 0.0308, 0.1971)	(-1.0401, -0.8337)	(s5.1122, s1.5119)	(s2.6834, s2.4046)
(0.4, 0.7)	(0.3140, 0.3411, 0.3449)	(0.0959, 0.7857, 0.1184)	(-1.0401, -0.7197)	(s3.2296, s1.2813)	(s3.6428, s2.2754)
(0.4, 0.9)	(0.3140, 0.3411, 0.3449)	(0.1331, 0.6836, 0.1833)	(-1.0401, -0.6561)	(s3.4397, s1.2984)	(s3.4971, s2.2919)
(0.7, 0.1)	(0, 0.8160, 0.1840)	(0.5267, 0.2205, 0.2529)	(-0.8286, -0.8995)	(s5.3115, s1.5980)	(s2.5002, s2.6133)
(0.7, 0.4)	(0, 0.8160, 0.1840)	(0.7721, 0.0308, 0.1971)	(-0.8286, -0.8337)	(s5.6323, s1.7083)	(s2.2960, s2.9703)
(0.7, 0.7)	(0, 0.8160, 0.1840)	(0.0959, 0.7857, 0.1184)	(-0.8286, -0.7197)	(s4.0297, s1.6341)	(s2.9869, s2.1070)
(0.7, 0.9)	(0, 0.8160, 0.1840)	(0.1331, 0.6836, 0.1833)	(-0.8286, -0.6561)	(s4.3004, s1.5805)	(s2.9212, s2.1554)
(0.9, 0.1)	(0, 0.9487, 0.0513)	(0.5267, 0.2205, 0.2529)	(-0.7319, -0.8995)	(s5.4637, s1.6326)	(s2.4467, s2.6745)
(0.9, 0.4)	(0, 0.9487, 0.0513)	(0.7721, 0.0308, 0.1971)	(-0.7319, -1.0141)	(s5.7691, s1.7137)	(s2.2239, s3.1425)
(0.9, 0.7)	(0, 0.9487, 0.0513)	(0.0959, 0.7857, 0.1184)	(-0.7319, -0.7197)	(s4.3692, s1.7662)	(s2.9124, s2.1042)
(0.9, 0.9)	(0, 0.9487, 0.0513)	(0.1331, 0.6836, 0.1833)	(-0.7319, -0.6561)	(s4.5888, s1.6892)	(s2.8636, s2.1477)